

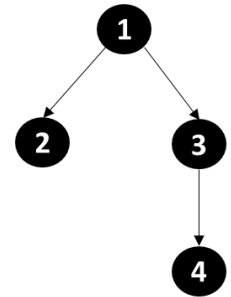
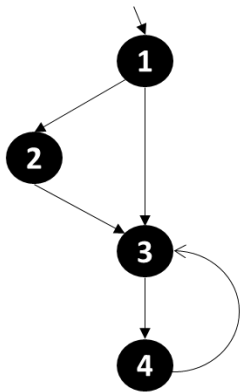
# Program Analysis

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“The theory of strings with concatenation has been widely argued as the basis of constraint solving for verifying string-manipulating programs. However, this theory is far from adequate for expressing many string constraints that are also needed in practice...”

– Chen et al., POPL 2018.

# Boeing 737 MAX Crash: Was Software To Blame And What Comes Next

Satellite data gathered from the Ethiopian Airlines flight and evidence from the crash site showed similarities with the Lion Air accident in Indonesia, which prompted the US Federal Aviation Administration to ground all Boeing MAX jets in service.

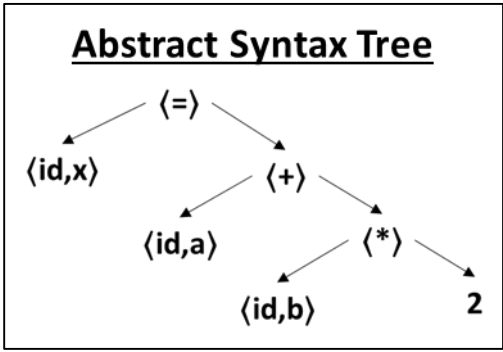
World | Reuters | Updated: March 15, 2019 10:59 IST



# Quick Review – Program Representations

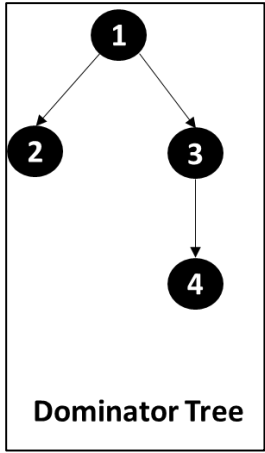
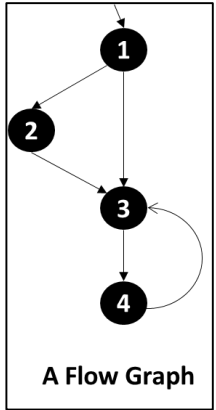
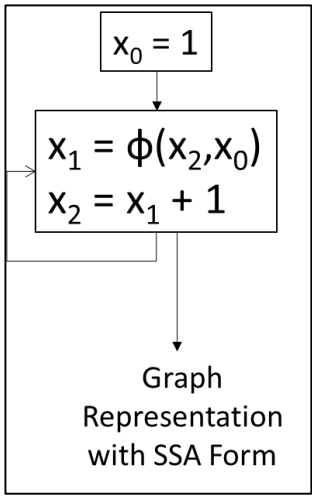
**Lexemes**  
`x = a + 2*b`

**Tokens**  
 [(identifier, x),  
 (operator, =),  
 (identifier, a),  
 (operator, +), (identifier, b),  
 (operator, \*),  
 (literal, 2), (separator, ;)]



**Three Address Code**

`x+y*z`  $\longrightarrow$  `t1 = y * z`  
`t2 = x + t1`



**Control Dependence**

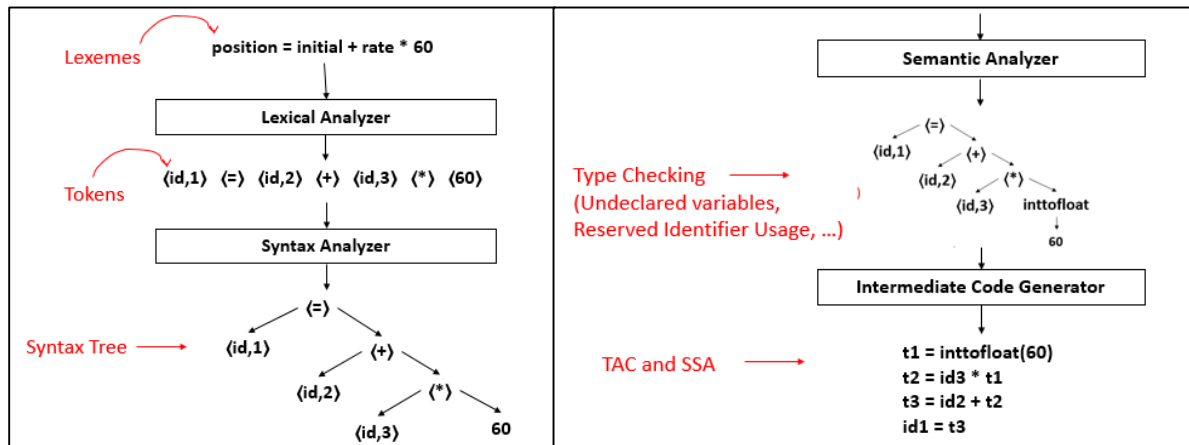
```

[x = 10]1;
if [(x > k)]2
  [y = 10]3;
else
  [y = 1]4;
[print y]5;
    
```

**Tools**  
 Eclipse JDT  
 Soot  
 JProfiler  
 Z3

**Runtime Structures**

# The Optimization Phase



*Parsed*

*No Compilation Errors!*

**Intermediate Code  
Generation**

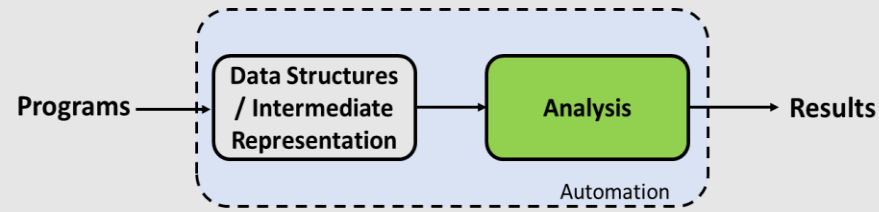


**Optimization**



**Code Generation**  
*Compiled*

**Now, we shall focus on optimization.**

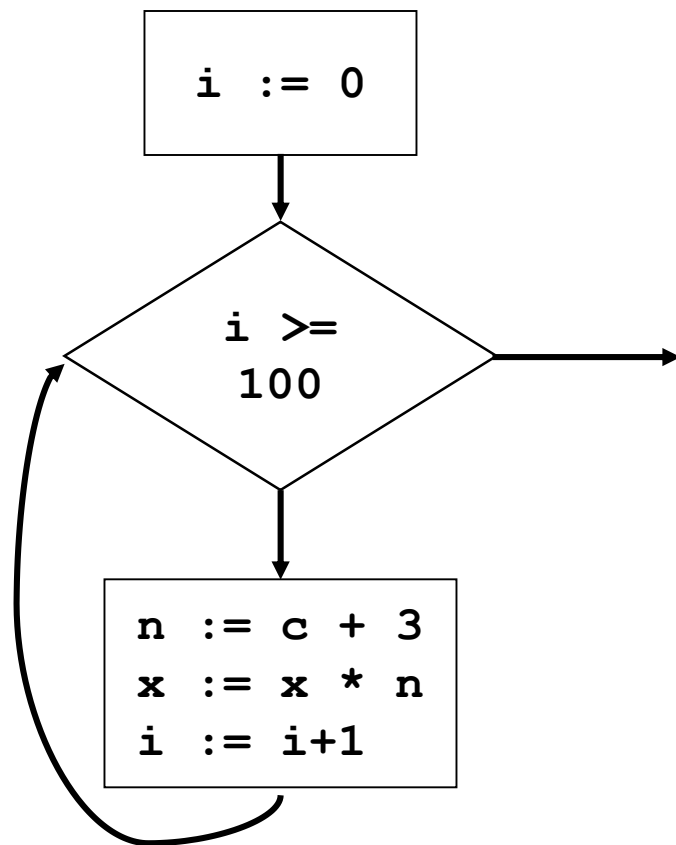


# Static Analysis

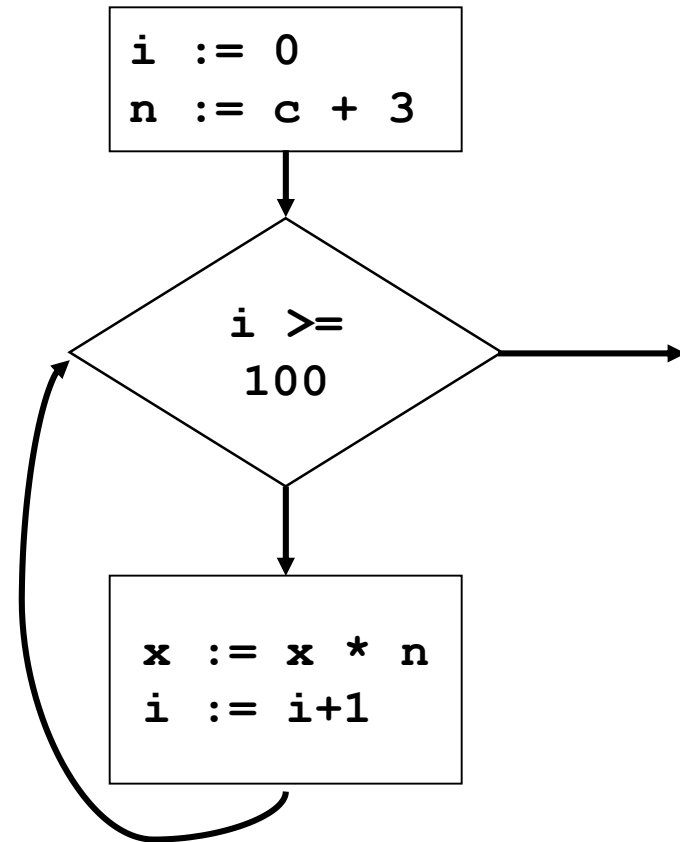
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## Overview

# Loop Invariant Code Motion



**Before Optimization**



**After Optimization**

# Code Hoisting

```
void fun (int x, int y)
{
  for (int i = 0; i < 100; i++)
  {
    int temp = x + y;
    System.out.println(temp);
  }
}
```

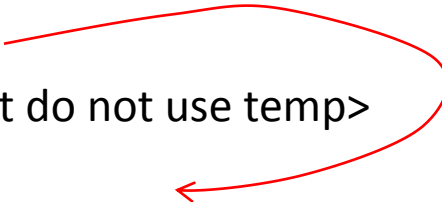
This line can be moved upwards  
(Code Hoisting)

**Can you come up with an example for code sinking?  
We need to move the code after the loop.**

# Code Sinking

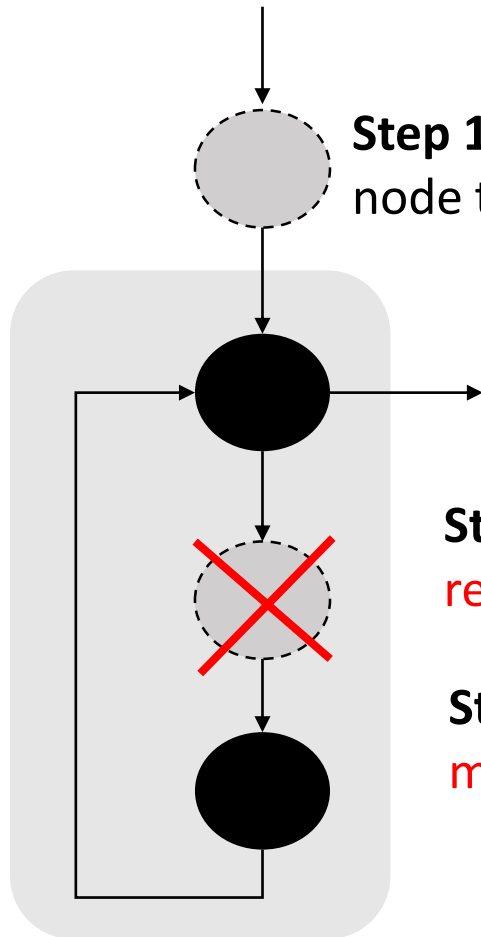
```
void fun (int x, int y)
{
  for (int i = 0; i < 100; i++)
  {
    int temp = x + y;
    ...<more lines that do not use temp>
  }
  System.out.println(temp);
}
```

This line can be moved downwards  
(Code Sinking)





# A Code Hoisting Algorithm



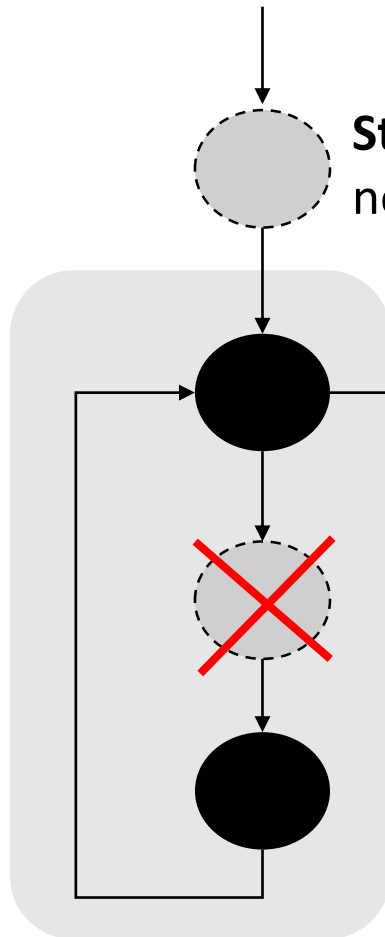
**Step 1:** Insert a **pre-header** node to hoist code.



**Step 2:** Find expressions whose operands have **reaching definitions** from outside the loop.

**Step 3:** Remove such statements and **move them** to the pre-header node.

# A Code Hoisting Algorithm



**Step 1:** Insert a **pre-header** node to hoist code.

**Step 2:** Find expressions whose operands have **reaching definitions** from outside the loop, or are constants. Mark them as loop invariant.

**Step 2.5:** Operands have exactly one **reaching definition** and that definition is loop invariant.

**Step 3:** Remove such statements (S) and **move them** to the pre-header node.


```
for(..) {  
  ...no use/def of a  
  a = 2;  
  temp = a + b;  
  ...no use/def of a  
}
```



# Beware of Function Calls

```
void fun (int x, int y)
{
  ...
  for (int i=0; i<=10; i++) {
    ...
    int value = rand.nextInt(50);
    ...
  }
  ...
}
```

Same method.  
Same parameters.  
Different results.  
So, we cannot move such lines.





Compilers  
Know  
Math!

# Constant Folding

`x := 3 + 5` → `x := 8`

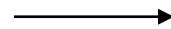
`x := "CMI" + ", Chennai"` → `x := "CMI, Chennai"`

They can do string operations too!

# Constant Propagation

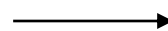
Dead Code  
Elimination can  
remove a and b.

```
a := 3
b := 5
x := a + b
```



```
a := 3
b := 5
x := 8
```

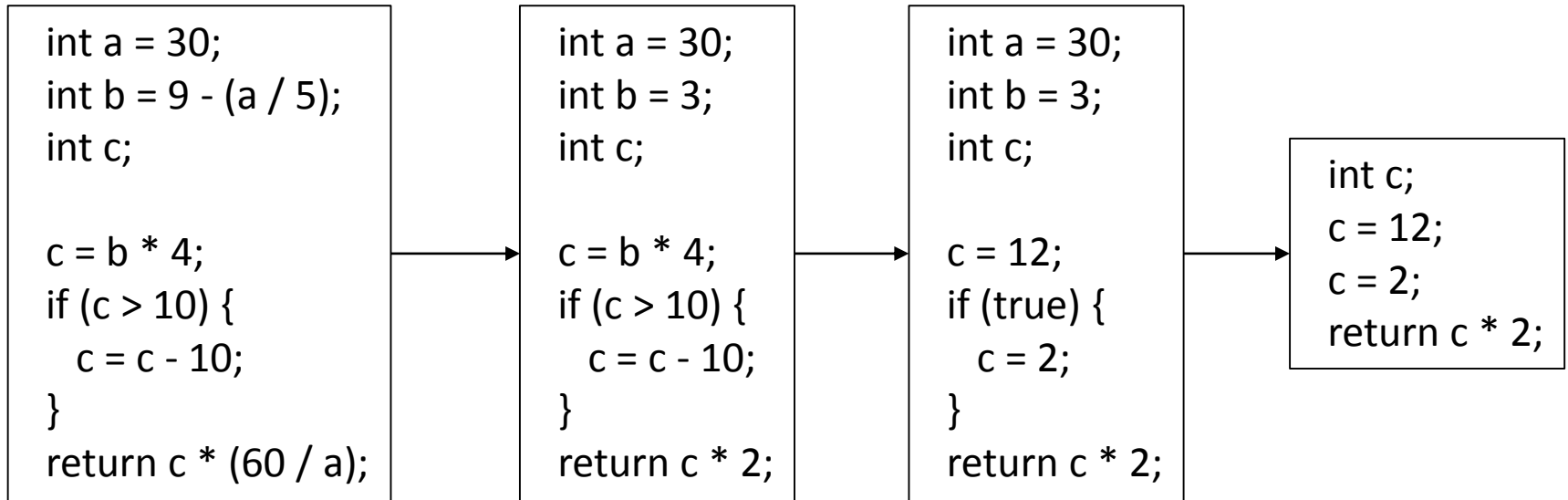
```
a := "CMI"
b := ", Chennai"
x := a + b
```



```
a := "CMI"
b := ", Chennai"
x := "CMI, Chennai"
```

Substituting the constant value to a variable

# Constant Folding and Propagation

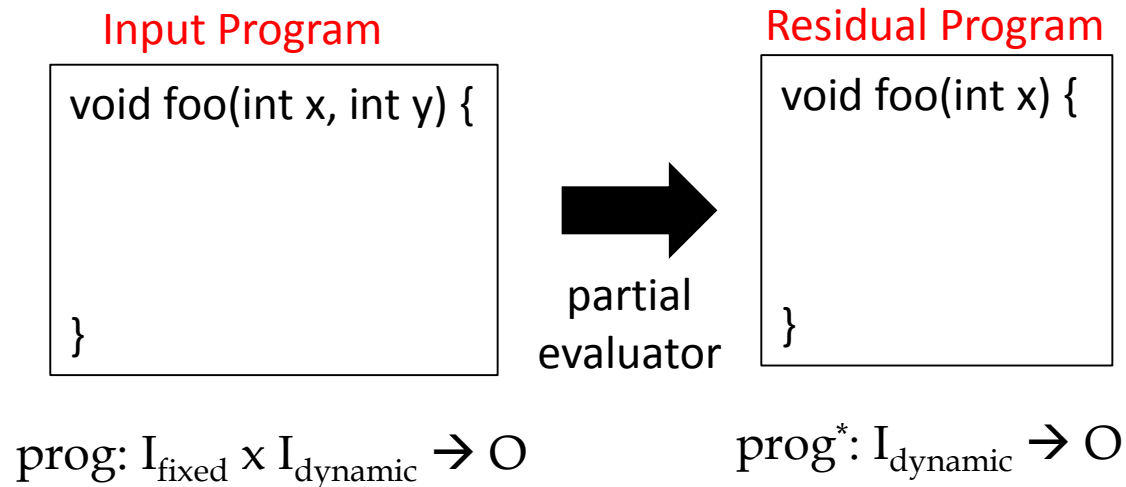


return 4;



# Partial Evaluation

Let us say,  $y$  is known to be fixed (a.k.a., configured) every time we execute this program.



where  $I$  and  $O$  are sets of inputs and Outputs respectively.

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[Partial Evaluation and Automatic Program Generation](#) (Book is freely available online)

Kleene's Smn Theorem

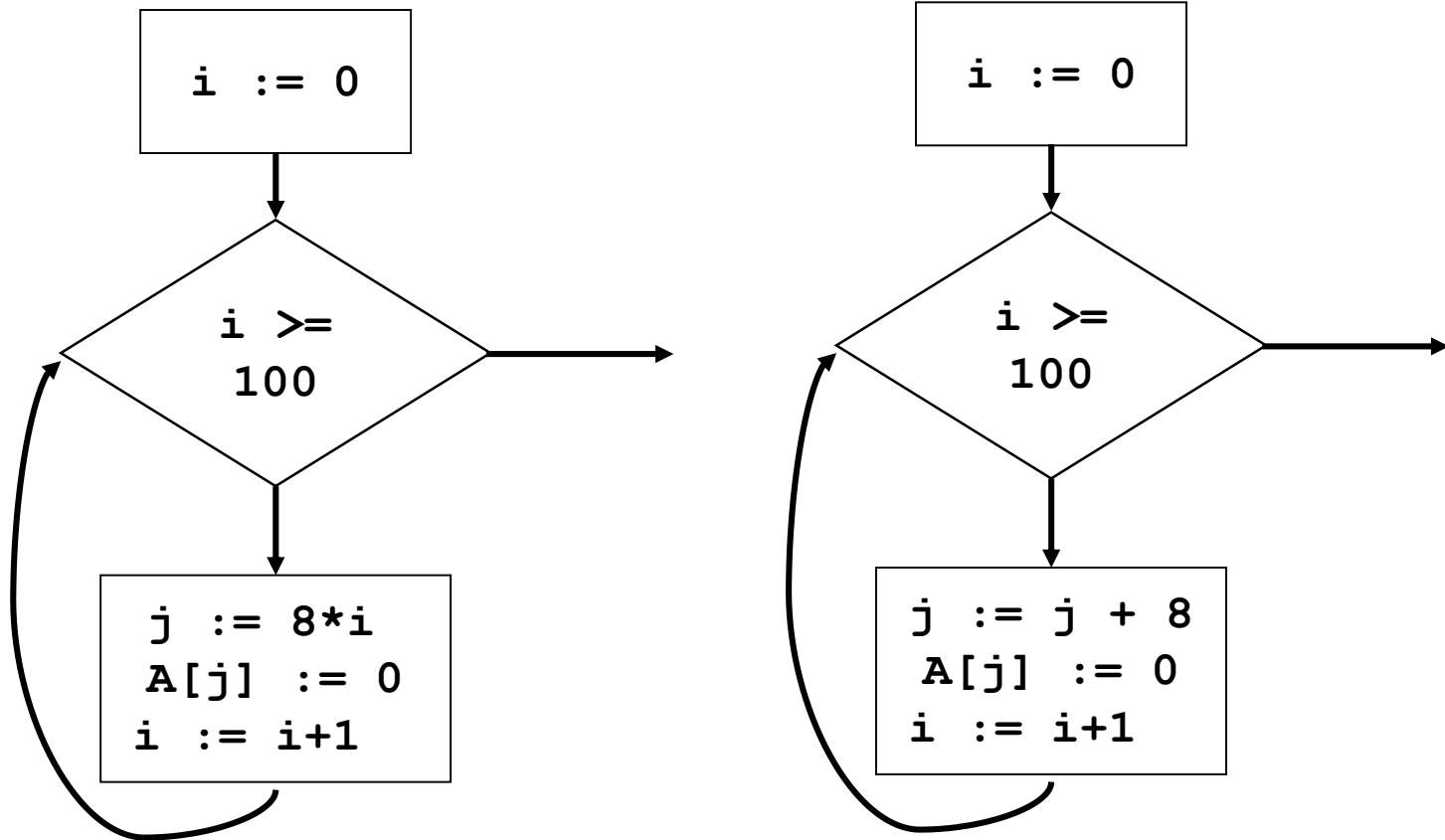


# Cross Product

- What is the cross product of two sets,  $A = \{a,b,c\}$  and  $Z = \{1,2,3\}$ ?
  - $A \times Z = ?$



# Strength Reduction



+ is less expensive than \*

# Graded Quiz

- Optimize the following snippet

```
void foo(int x, int y) {  
    y = x * 15;  
    System.out.println(y);  
}
```



# Peephole Optimization

Before	After
$y = x / 8$	$y = x \gg 3$
$y = x * 64$	$y = x \ll 6$
$y = x * 2$	$y = x \ll 1$

# More Examples

- Can you find the improvement?

```
int factorial(int i)
{
    if (i == 0)
        return 1;
    else
        return i *
            factorial(i - 1);
}
```

```
static int factorialTable[] =
    {1, 1, 2, 6, 24, 120, 720 /*
    etc */};

int factorial(int i)
{
    return factorialTable[i];
}
```

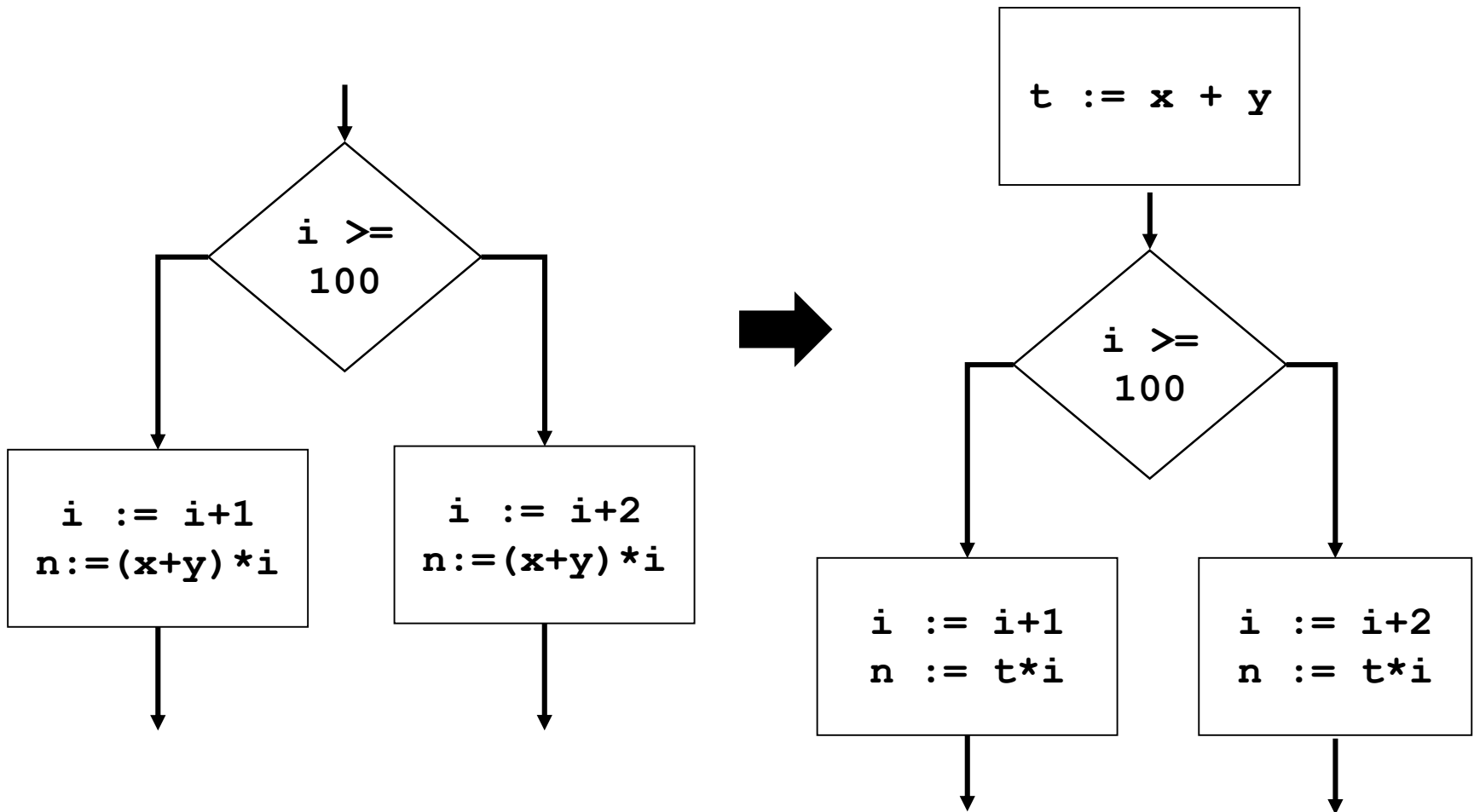
# More Examples

- Can you find a variant?

```
int foo(int n, int r) {
    if (n > 0) {
        return (n%r + foo (n/r, r ));
    }
    else return 0;
}

void main() {
    printf("%d ", foo(346,10));
}
```

# Common Subexpressions



# More Examples

- What could be improved here?

```
float x,y,a,b,c,d;  
...  
x = (a/b)*c;  
y = (a/b)*d;
```

- Common Sub-Expression (a/b) can be computed only once.



# Source Code Optimization

- How to optimize this?

```
if (x != 5) x = 5;
```

- Simply,

```
x = 5;
```

# Beyond Compiler Optimizations

---

# Applications

- Apart from bug detection and optimization, static program analysis techniques are used in
  - Source Code Plagiarism Detection (Moss, ...)
  - Clone Detection
  - Code Search
  - Automated MOOC Assignment Feedback
  - Automated Programming Quiz
  - Program Translators
  - Code Completion
  - Refactoring
  - and so on...

# Readings

- **Before the next lecture, please read**
  - Chapters 1 and 2 of Anders Møller and Michael I. Schwartzbach's book on Static Program Analysis.

# Data Flow Analysis

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The Classic Four!

# WHILE Language

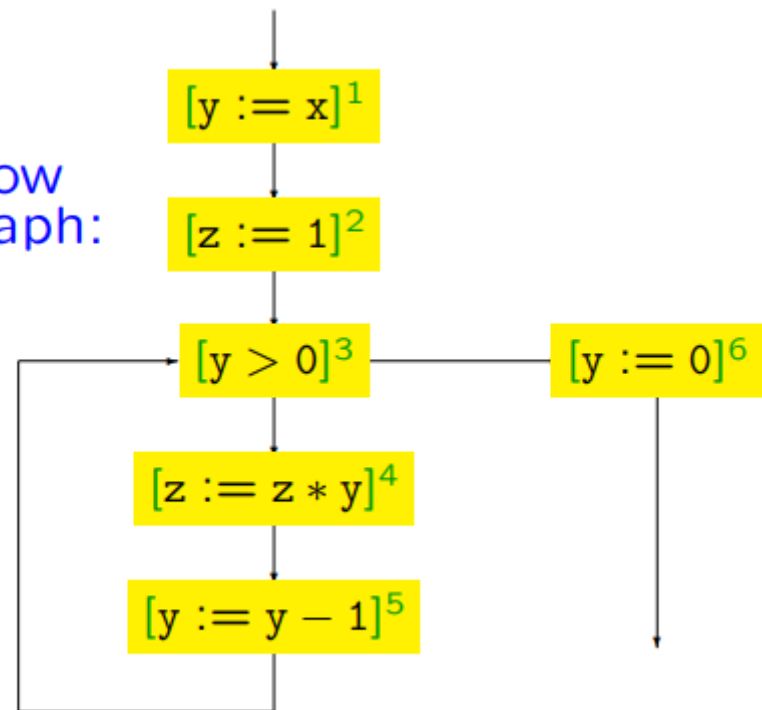
- Simple Imperative Language
- S refers to Statements, a is an Arithmetic Expression and b is a Boolean Expression

$$a ::= x \mid n \mid a_1 \text{ op}_a a_2$$
$$b ::= \text{true} \mid \text{false} \mid \text{not } b \mid b_1 \text{ op}_b b_2 \mid a_1 \text{ op}_r a_2$$
$$S ::= [x := a]^! \mid [\text{skip}]^! \mid S_1; S_2 \mid$$
$$\text{if } [b]^! \text{ then } S_1 \text{ else } S_2 \mid \text{while } [b]^! \text{ do } S$$

# Labeled Programs and Control Flow

```
[y := x]1;  
[z := 1]2;  
while [y > 0]3 do  
  [z := z * y]4;  
  [y := y - 1]5  
od;  
[y := 0]6
```

Flow graph:



# Reaching Definitions

---



# Reaching Definitions (RD) Analysis

- The assignment  $[x := a]^{\ell}$  reaches  $\ell'$  if there is an execution where  $x$  was last assigned at  $\ell$ .

```
[y = x]1;  
[z = 1]2;  
while [(y > 0)]3 {  
    [z = z * y]4;  
    [y = y - 1]5;  
}  
[y = 0]6
```

Does  $[z := 1]^2$  reach 5?

# Notations

```
[y := x]1;  
[z := 1]2;  
while [y > 0]3 do  
    [z := z * y]4;  
    [y := y - 1]5  
od;  
[y := 0]6
```

```
[y = x]1;  
[z = 1]2;  
while [(y > 0)]3 {  
    [z = z * y]4;  
    [y = y - 1]5;  
}  
[y = 0]6
```

# RD Analysis

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$(x, ?), (y, ?), (z, ?)$	$(x, ?), (y, 1), (z, ?)$
2		
3		
4		
5		
6		

## Labeled Input Program

```
[y = x]1;  
[z = 1]2;  
while [(y > 0)]3 {  
    [z = z * y]4;  
    [y = y - 1]5;  
}  
[y = 0]6
```

# RD Analysis

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$(x, ?), (y, ?), (z, ?)$	$(x, ?), (y, 1), (z, ?)$
2	$(x, ?), (y, 1), (z, ?)$	$(x, ?), (y, 1), (z, 2)$
3		
4		
5		
6		

## Labeled Input Program

```
[y = x]1;  
[z = 1]2;  
while [(y > 0)]3 {  
    [z = z * y]4;  
    [y = y - 1]5;  
}  
[y = 0]6
```

# RD Analysis

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$(x, ?), (y, ?), (z, ?)$	$(x, ?), (y, 1), (z, ?)$
2	$(x, ?), (y, 1), (z, ?)$	$(x, ?), (y, 1), (z, 2)$
3	$(x, ?), (y, 1), (z, 2)$ $(z, 4), (y, 5)$	$(x, ?), (y, 1), (z, 2), (z, 4)$ $, (y, 5)$
4		
5		
6		

## Labeled Input Program

```
[y = x]1;  
[z = 1]2;  
while [(y > 0)]3 {  
    [z = z * y]4;  
    [y = y - 1]5;  
}  
[y = 0]6
```

# RD Analysis

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$(x, ?), (y, ?), (z, ?)$	$(x, ?), (y, 1), (z, ?)$
2	$(x, ?), (y, 1), (z, ?)$	$(x, ?), (y, 1), (z, 2)$
3	$(x, ?), (y, 1), (z, 2)(z, 4), (y, 5)$	$(x, ?), (y, 1), (z, 2), (z, 4), (y, 5)$
4	$(x, ?), (y, 1), (z, 2)(z, 4), (y, 5)$	$(x, ?), (y, 1), (z, 4), (y, 5)$
5	$(x, ?), (y, 1), (z, 4), (y, 5)$	$(x, ?), (y, 5), (z, 4)$
6	$(x, ?), (y, 1), (z, 2), (z, 4), (y, 5)$	$(x, ?), (y, 6), (z, 2), (z, 4)$

## Labeled Input Program

```
[y = x]1;  
[z = 1]2;  
while [(y > 0)]3 {  
    [z = z * y]4;  
    [y = y - 1]5;  
}  
[y = 0]6
```

# How to Automate?

- We write a system of equations

$$RD_{\text{exit}}(1) = (RD_{\text{entry}}(1) \setminus \{ (y,\ell) \mid \ell \in \mathbf{Lab} \} ) \cup \{ (y,1) \}$$

$$RD_{\text{exit}}(2) = (RD_{\text{entry}}(2) \setminus \{ (z,\ell) \mid \ell \in \mathbf{Lab} \} ) \cup \{ (z,2) \}$$

$$RD_{\text{exit}}(3) = RD_{\text{entry}}(3)$$

$$RD_{\text{exit}}(4) = (RD_{\text{entry}}(4) \setminus \{ (z,\ell) \mid \ell \in \mathbf{Lab} \} ) \cup \{ (z,4) \}$$

$$RD_{\text{exit}}(5) = (RD_{\text{entry}}(5) \setminus \{ (y,\ell) \mid \ell \in \mathbf{Lab} \} ) \cup \{ (y,5) \}$$

$$RD_{\text{exit}}(6) = (RD_{\text{entry}}(6) \setminus \{ (y,\ell) \mid \ell \in \mathbf{Lab} \} ) \cup \{ (y,6) \}$$

where  $\mathbf{Lab} = \{1,2,3,4,5,6\}$

# System of Equations...

- Similarly, specify  $RD_{\text{entry}}(\ell)$  for each line.

$$RD_{\text{entry}}(2) = RD_{\text{exit}}(1)$$

$$RD_{\text{entry}}(3) = RD_{\text{exit}}(2) \cup RD_{\text{exit}}(5)$$

$$RD_{\text{entry}}(4) = RD_{\text{exit}}(3)$$

$$RD_{\text{entry}}(5) = RD_{\text{exit}}(4)$$

$$RD_{\text{entry}}(6) = RD_{\text{exit}}(3)$$

$$RD_{\text{entry}}(1) = \{(x, ?), (y, ?), (z, ?)\}$$



# System of Equations...

- 12 Equations with 11 unknowns

**Find the least solution**

- We have a 12-Tuple,  $\overrightarrow{RD} = RD_{\text{entry}}(1), \dots, RD_{\text{exit}}(6)$
- $\overrightarrow{RD} = F(RD)$

# A Simple Iterative Algorithm

$\overrightarrow{RD} = (\emptyset, \dots, \emptyset)$

$j = 0;$

while  $\overrightarrow{RD} \neq F(RD_1, \dots, RD_{12})$

do  $\overrightarrow{RD} := F(RD_1, \dots, RD_{12})$

# Simple Iteration

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$\emptyset$	$\emptyset$
2	$\emptyset$	$\emptyset$
3	$\emptyset$	$\emptyset$
4	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$

**F(RD)**  
→

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$\{(x,?), (y,?), (z,?)\}$	$\emptyset$
2	$\emptyset$	$\emptyset$
3	$\emptyset$	$\emptyset$
4	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$

# F(RD)

$$RD_{\text{entry}}(1) = \{(x,?), (y,?), (z,?)\}$$

$$RD_{\text{entry}}(2) = RD_{\text{exit}}(1)$$

$$RD_{\text{entry}}(3) = RD_{\text{exit}}(2) \cup RD_{\text{exit}}(5)$$

$$RD_{\text{entry}}(4) = RD_{\text{exit}}(3)$$

$$RD_{\text{entry}}(5) = RD_{\text{exit}}(4)$$

$$RD_{\text{entry}}(6) = RD_{\text{exit}}(3)$$

$$RD_{\text{exit}}(1) = (RD_{\text{entry}}(1) \setminus \{(y,\ell) \mid \ell \in \mathbf{Lab}\}) \cup \{(y,1)\}$$

$$RD_{\text{exit}}(2) = (RD_{\text{entry}}(2) \setminus \{(z,\ell) \mid \ell \in \mathbf{Lab}\}) \cup \{(z,2)\}$$

$$RD_{\text{exit}}(3) = RD_{\text{entry}}(3)$$

$$RD_{\text{exit}}(4) = (RD_{\text{entry}}(4) \setminus \{(z,\ell) \mid \ell \in \mathbf{Lab}\}) \cup \{(z,4)\}$$

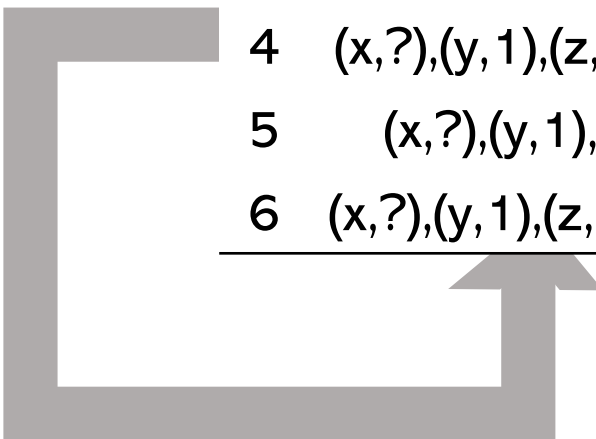
$$RD_{\text{exit}}(5) = (RD_{\text{entry}}(5) \setminus \{(y,\ell) \mid \ell \in \mathbf{Lab}\}) \cup \{(y,5)\}$$

$$RD_{\text{exit}}(6) = (RD_{\text{entry}}(6) \setminus \{(y,\ell) \mid \ell \in \mathbf{Lab}\}) \cup \{(y,6)\}$$

# Reaches a Least Fixed Point

$\ell$	$RD_{\text{entry}}(\ell)$	$RD_{\text{exit}}(\ell)$
1	$(x, ?), (y, ?), (z, ?)$	$(x, ?), (y, 1), (z, ?)$
2	$(x, ?), (y, 1), (z, ?)$	$(x, ?), (y, 1), (z, 2)$
3	$(x, ?), (y, 1), (z, 2)(z, 4), (y, 5)$	$(x, ?), (y, 1), (z, 2), (z, 4), (y, 5)$
4	$(x, ?), (y, 1), (z, 2)(z, 4), (y, 5)$	$(x, ?), (y, 1), (z, 4), (y, 5)$
5	$(x, ?), (y, 1), (z, 4), (y, 5)$	$(x, ?), (y, 5), (z, 4)$
6	$(x, ?), (y, 1), (z, 2), (z, 4), (y, 5)$	$(x, ?), (y, 6), (z, 2), (z, 4)$

**F(RD)**



# The Question

- Does the definition of  $z$  in line 2 reach line 5?

$[y := x]^1;$

$[z := 1]^2;$

while  $[y > 0]^3$  do

$[z := z * y]^4;$

$[y := y - 1]^5$

od;

$[y := 0]^6$

Answer: No!

Since,  $RD_{\text{entry}}(5) = (x, ?), (y, 1), (z, 4), (y, 5)$

There is no  $(z, 2)$  in it.